

Segmenting Colour Images on the Basis of a Fuzzy Hierarchical Approach

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Abstract

In this paper we deal with two problems related to imprecision in colour image segmentation processes: to decide whether a set of pixels verify the property "to be homogeneously coloured", and to represent the set of possible segmentations of an image at different precision levels. In order to solve the first problem we introduce a measure of distance between colours in the CIE $L^*a^*b^*$ space, that allows us to measure the degree of homogeneity of two pixels p and q on the basis of the maximum distance between the colours of consecutive pairs of pixels in any path linking p and q . Since homogeneity is a matter of degree, we define a (fuzzy) segmentation of an image as a set of fuzzy regions, each of them being a fuzzy subset of pixels, that we obtain by using a region growing technique. The membership degree of each pixel to each region is calculated on the basis of our homogeneity measure. The second problem is solved by introducing a fuzzy similarity relation between the fuzzy regions in this initial segmentation. The different α -cuts of the similarity relation define the set of precision levels, from which a nested hierarchy of fuzzy segmentations is finally obtained.

Keyword: Colour image segmentation, fuzzy segmentation, hierarchical segmentation, colour distance.

1 Introduction

As it is well known, many image analysis techniques take as starting point a segmentation of the image, that is, a partition of the set of pixels in the image into connected subsets (called regions) on the basis of a certain criterion, that in most cases is color homogeneity of the pixels in regions.

Many types of segmentation techniques have been proposed in the literature, each of them based on a certain methodology to calculate the regions. A first group of segmentation methods, pixel based methods, consider a region as a set of pixels satisfying a class membership function. Among them, histogram based

techniques [6] and clustering algorithms are significative examples [17]. A second kind of segmentation methods, area based techniques, defines regions as a set of pixels verifying an uniformity condition, as occurs in regions growing techniques [10] and split and merge algorithms [1]. A different point of view about regions consists on describing them as a set of pixels bounded by a colour contour, as edge based algorithms do.

Most of the proposals falling in the aforementioned categories provide a crisp segmentation of images, where each pixel has to belong to an unique region. However, separation between regions is usually imprecise in natural images, as can be noticed in shadows, brights and color gradients, so crisp techniques are not often appropriate. To solve this problem, some approaches propose the definition of *region* as a fuzzy subset of pixels, in such a way that every pixel of the image has a membership degree to that region [2].

The majority of these fuzzy techniques are based on fuzzy clustering [4], like C-means algorithms, which defines a set of centroids and compute the membership value for all the pixels in the image to each centroid [14] [16]. Another example of fuzzy techniques are those based on the definition of fuzzy color histograms, where every pixel in the image belongs to every histogram's bin in a certain degree [8], or fuzzy homogeneity histograms [5]. Less extended ideas for fuzzy segmentation are the ones based on different fuzzy resources, like using If-Then rules to detect borders [5] [7].

A drawback of most of these approaches is that they don't take into account one of the characteristics of regions, i.e., they must be topologically connected. As a consequence, pixels belonging to separate and different regions, could be assigned to the same cluster. Hence, we consider that spatial information related to adjacency between regions should be introduced in order to overcome this situation. For this purpose, an interesting approach is to incorporate information about the topology of the image [10].

Another important problem related to image segmentation is that of granularity of the resulting fuzzy partition. Granularity issues are inherently present in image segmentation since number, size, and homogeneity of regions depend on the precision level we consider. Let's illustrate this idea with the image in figure 1. At a high precision level we could consider several regions into the turtle's shell, corresponding to areas with different brown colours. However, at lower granularity levels we could consider an unique region corresponding to the whole turtle. In general, when high precision is employed we obtain "many", small, and homogeneous regions. As the precision level is diminished, less, bigger, more heterogeneous regions are obtained by joining together several regions of previous levels. At the bottom, we have one single region that covers the whole image.

Indirectly, some of these techniques [9] supply results that may be considered a solution to take into account these different levels of detail, since in every step of the algorithm the two most similar regions are joint into a single one. Nevertheless they only use information about the center of the regions like their average colour, so they don't consider how soft or abrupt the transition between adjacent regions is.

To face the problems described above, a hierarchical approach to fuzzy segmen-



Figure 1: A real image example that can be viewed with different detail levels.

tation of colour images is proposed in this paper. The methodology works in two steps:

- In the first one, introduced in [3], a collection of fuzzy regions is calculated by using the region growing approach. In this stage, a region is defined as a fuzzy subset of connected pixels and it is constructed using topographic and colour information, i.e., two pixels will be assigned to the same region if they are connected through a path of similar colours. A distance defined in the $CIE(L^*a^*b^*)$ colour space is used in the growth of a region both (i) to select the pixels which will be linked in each step of the algorithm, and (ii) to calculate the membership degree of each point to each region.
- In the second step, a nested hierarchy of fuzzy partitions is calculated on the basis of a max-min transitive similarity relation between regions. The set of levels of the hierarchy corresponds to the different significant α -cuts of the relation. The similarity relation is calculated on the basis of a resemblance one (degree of compatibility of regions) and using the idea of path between regions. This technique, that incorporates information about the aforementioned characteristics of the transition between regions, is coherent with the one employed between pixels in the previous stage.

The rest of the paper is organized as follows: in section 2 the colour information processing used in our approach is described. Section 3 summarizes the proposed methodology to extract the collection of fuzzy regions (first stage). Section 4 describes in detail the proposed hierarchical union of regions (second stage). Finally, some results and the main conclusions are showed in sections 5 and 6 respectively.

2 Colour processing

An important aspect to take into account in image segmentation is the colour information processing. The most common solutions in the literature are (i) combining the information of each band into a single value before processing (for example, the gradient), or (ii) analyzing each band separately and then combining the results (for example, histogram analysis of each band and subsequent combination). Apart from the difficulty to choose an adequate criterion to pool the data, also appears the problem of the fuzziness associated with the own definition of the color spaces,

which some techniques like [11] try to solve by modeling the human perceptual system. However there is still a problem to work out in above mentioned solutions: they apply the same combination rule to the whole image, without considering the particularities that appear in the comparison of two colours. That problem is more significant in methods, like those based on region growing, where the decision in each step depends on the difference between pixels.

To process the colour information, we propose a methodology based on a distance between colours defined within a perceptual color space. Although the RGB is the most used model to acquire digital images, it is well known that it is not adequate for colour image segmentation. Instead, other colour spaces based on human perception (HSI, HSV, $CIE(L^*a^*b^*)$, etc.) seem to be a better choice for this purpose [12].

In this paper, the $CIE(L^*a^*b^*)$ colour space is used. This colour model has been developed on the basis of observer's judgements and it provides a measure of the perceived difference between colours. Specifically, it is based on Munsell's uniform space [13] and it is given by the quantities L^* , a^* and b^* defined as:

$$\begin{aligned} L^* &= 116f\left(\frac{Y}{Y_n}\right) - 16 \\ a^* &= 500\left[f\left(\frac{X}{X_n}\right) - f\left(\frac{Y}{Y_n}\right)\right] \\ b^* &= 200\left[f\left(\frac{Y}{Y_n}\right) - f\left(\frac{Z}{Z_n}\right)\right] \end{aligned} \quad (1)$$

where

$$f(x) = \begin{cases} x^{\frac{1}{3}} & \text{if } x > 0.008856 \\ 7.787x + \frac{16}{116} & \text{otherwise} \end{cases} \quad (2)$$

with X , Y and Z being the tristimulus coordinates of the represented colour, and X_n , Y_n and Z_n the tristimulus values of the white colour [13]. In this space, the L^* value is a measure of the lightness, while a^* and b^* define together the hue and saturation of the color. Specifically, the a^* axis runs from red to green, and the b^* axis from yellow to blue.

Distance between colours

In order to determine how different are two given colours we propose the following measure:

Definition 2.1 *The difference between two colour stimuli p and q , each given in terms of L^* , a^* , b^* , is:*

$$\Delta C^*(p, q) = \sqrt{(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2} \quad (3)$$

For the sake of simplicity, we have left out the spatial parameters (x, y) in the notation $p(x, y)$. This Euclidean distance within the $CIE(L^*a^*b^*)$ colour space is related to the perceptual difference measured by human observers [13].

3 Extracting initial fuzzy regions

In this section, a method to obtain an initial set of fuzzy regions, noted as $\tilde{\Theta} = \{\tilde{R}_1, \tilde{R}_2, \dots, \tilde{R}_m\}$, is proposed. Firstly, a technique to calculate a collection of seed points, noted as $\Theta = \{r_1, r_2, \dots, r_m\}$, is outlined in section 3.1. Secondly, a method to fuzzify that set of seed points is proposed. For this purpose, a membership function for fuzzy regions is defined (section 3.3) based on a distance between pixels (section 3.2).

3.1 Seed points

In order to select areas that hopefully may correspond to structural units, seed regions will be located in the local minima of the image defined by the equation

$$\Delta I(p) = \min \{ \Delta C^*(p, q), q \in W_r(p) \}$$

where $W_r(p)$ represents the neighborhood of p defined as the set of pixels contained in a window of size $r \times r$ centered at p . Let us remark that ΔI , and consequently the seed point selection method, is defined on the basis of the colour-difference formula ΔC^* (equation 3) within the $CIE(L^*a^*b^*)$ colour space. In this paper, r has been fixed to 3 and the local minima have been calculated over a window of size 7×7 .

It would be desirable for each *object* in the image to contain a single seed, but this is not often the case. Indeed, it is usual to find several seeds into the same *object* (which implies an overload of regions). To solve this problem, some authors propose to discard seeds during the segmentation process on the basis of some criterion. Apart from the difficulty to choose that criterion, this solution could eliminate seeds corresponding to zones of interest. In general, the main problem is that the number and size of regions depend on the precision level we consider when looking at images. As we shall show in section 4, we face this problem on the basis of a hierarchical union of regions.

3.2 Distance between pixels

Our target now is to define a measure of how different are two pixels, taking into account not only their colour, but also including information about the topology of the area separating them. For this purpose we use information about the path joining them, having a path defined as follows:

Definition 3.1 *A path between two pixels p and q is a sequence*

$$\pi_{pq} = (r_0, r_1, \dots, r_k)$$

where $k \geq 1$ such that $r_0 = p$ and $r_k = q$ and r_i is connected to $r_{i+1} \forall i \in \{0, \dots, k-1\}$.

We will note \prod_{pq} to the set of possibles paths linking the pixels p and q through pixels of the image I as \prod_{pq} .

Definition 3.2 The cost of a given a path $\pi_{pq} \in \Pi_{pq}$, is defined as the greatest distance between two consecutive points on the path:

$$\text{cost}(\pi_{pq}) = \max \{ \Delta C^*(r_i, r_{i+1}) / r_i, r_{i+1} \in \pi_{pq} \} \quad (4)$$

where r_i and r_{i+1} are two consecutive points of π_{pq} , and ΔC^* is defined in equation (3). Taking it into account, we can define the optimum path between p and q as the path that links both points with minimum cost, in the following way:

Definition 3.3 The optimum path between p and q is:

$$\pi_{pq}^* = \underset{\pi_{pq} \in \Pi_{pq}}{\operatorname{argmin}} \{ \text{cost}(\pi_{pq}) \} \quad (5)$$

Based on this optimum path, we can get the measure of the distance between two pixels as shown in this definition:

Definition 3.4 The distance between two pixels p and q as the cost of the optimum path from p to q :

$$d(p, q) = \text{cost}(\pi_{pq}^*) \quad (6)$$

Let us remark that the distance defined in (6) uses topographic information (paths linking the pixels) and distances between colours. In addition, it is sensitive to the presence of edges in the following sense: if the optimum path linking two points p and q pass through an edge (that is, a point which separates two regions), its cost, and consequently the distance between p and q , will be increased. That is because of the fact that there are consecutive points, in the portion of the path that cross over the edge, with a high distance between them.

3.3 Membership function for fuzzy regions

Since we have fuzzy regions, it is necessary to define a measure that indicates the degree in which each pixel in the image belongs to each region. For this purpose, we introduce a membership function associated to each region, which definition is the following:

Definition 3.5 The membership degree $\mu_{\widetilde{R}_s}(p)$ of a pixel p to a fuzzy region \widetilde{R}_s is:

$$\mu_{\widetilde{R}_s}(p) = 1 - \frac{d(p, r_s)}{M} \quad (7)$$

where $r_s \in \Theta$ is the seed point of \widetilde{R}_s , and M is a normalization factor given by

$$M = \max \{ d(q, r_s), q \in I \} \quad (8)$$

Using equation (7) we can calculate the membership degree of every point $p \in I$ to each region \widetilde{R}_s . This allows us to obtain the set of fuzzy regions $\widetilde{\Theta} = \{ \widetilde{R}_1, \widetilde{R}_2, \dots, \widetilde{R}_m \}$ from the set of seed points $\Theta = \{ r_1, r_2, \dots, r_m \}$. An algorithm to calculate $\widetilde{\Theta}$ is proposed in [3] with a computational complexity of $O(mn)$, where n is the number of pixels of the image I and m is the number of seeds.

4 Hierarchical union of fuzzy regions

In the previous section we have described a methodology to get an initial fuzzy segmentation. In this section we are going to use a similar approach to provide a methodology to perform a hierarchical union of regions on the basis of their resemblance and a given precision degree.

4.1 Resemblance relation

Since our target is to join regions, it is necessary to introduce a measure to determine how similar they are, and therefore whether they can be joint or not. To obtain this measure, we introduce the following resemblance relation:

Definition 4.1 *The fuzzy resemblance relation $Res_{\tilde{\Theta}}$ between fuzzy regions in $\tilde{\Theta}$,*

$$Res_{\tilde{\Theta}} : \tilde{\Theta} \times \tilde{\Theta} \rightarrow [0, 1]$$

is

$$Res_{\tilde{\Theta}}(\widetilde{R_s}, \widetilde{R_t}) = \max_{p \in I} \{\min[\mu_{\widetilde{R_s}}(p), \mu_{\widetilde{R_t}}(p)]\} \quad (9)$$

Definition 4.2 *Connected regions: We say that two regions are connected iff $Res_{\tilde{\Theta}}(\widetilde{R_s}, \widetilde{R_t}) > 0$.*

This means that two regions are so similar as the highest membership value of the pixels belonging to both regions.

Since the value $Res_{\tilde{\Theta}}(\widetilde{R_s}, \widetilde{R_t})$ is a measure of the compatibility degree between $\widetilde{R_s}$ and $\widetilde{R_t}$, the resemblance between regions plays the same role as the distance between pixels, i.e., they give us information to make a decision about when to join them together into a single region. It is easy to show that $Res_{\tilde{\Theta}}$ is reflexive (since the membership function of every fuzzy region is normalized), and symmetric.

A limitation of this fuzzy relation is that it considers only those pixels that are in the intersection of the support of both fuzzy regions. However, as in the case of individual pixels, we should take into account the idea of continuity, in the sense that given two regions $\widetilde{R_s}$ and $\widetilde{R_t}$, we could find a chain of connected regions from $\widetilde{R_s}$ to $\widetilde{R_t}$ such that the minimum similarity between consecutive regions is greater than $Res_{\tilde{\Theta}}(\widetilde{R_s}, \widetilde{R_t})$.

An example is a gradation like that in figure 2. In such a case, assuming that the resemblance between connected regions is the same, it seems that there are only two possibilities to build a nested hierarchy: to join all the regions or to keep them all separate. Seeming natural, these decisions cannot be adopted if we use $Res_{\tilde{\Theta}}$. As a conclusion we think that, in general, we should take into account topographic information when determining the relation between regions.

4.2 Similarity relation

To comply with the notion of continuity, we have defined a similarity relation following the same idea used for distance between individual pixels, i.e., to find

paths of regions and to calculate the similarity between a given pair of regions as the minimum similarity of consecutive pairs of regions in any path linking them. So we first need to precise what a path between regions is:

Definition 4.3 A path between regions \widetilde{R}_s and \widetilde{R}_t is a sequence

$$\omega = (\widetilde{R}_{r_1}, \widetilde{R}_{r_2}, \dots, \widetilde{R}_{r_o})$$

with $o \geq 2$ such that $\widetilde{R}_{r_1} = \widetilde{R}_s$ and $\widetilde{R}_{r_o} = \widetilde{R}_t$, and \widetilde{R}_{r_k} is connected to $\widetilde{R}_{r_{k+1}}$ $\forall k \in \{1, \dots, o-1\}$.

Having a path between regions defined as a chain of consecutive and non repeated regions, we can get the set of all the possible path joining regions in the image, that we will note as $\Omega^{\widetilde{\Theta}}$, where $\widetilde{\Theta}$ is the set of all regions in the image. Following this notation $\Omega_{st}^{\widetilde{\Theta}} \subseteq \Omega^{\widetilde{\Theta}}$ will be the set of paths between regions \widetilde{R}_s and \widetilde{R}_t .

Since the definition of the similarity relation needs to select just one of all the paths joining two regions, we introduce a measure to compare all of them and find the optimum path: path's benefit, defined as follows:

Definition 4.4 The benefit of a path $\omega \in \Omega^{\widetilde{\Theta}}$ is

$$ben(\omega) = \min_{k \in \{1, \dots, o-1\}} Res_{\widetilde{\Theta}}(\widetilde{R}_{r_k}, \widetilde{R}_{r_{k+1}}) \quad (10)$$

This measure has a similar meaning to the cost defined for pixels, with the difference that benefit represents similarity, while cost indicates dissimilarity.

The benefit of a path is the similarity degree between the first and the last region of the path, as given by the sequence of similarities of connected regions. If the path is cyclic (i.e. $\exists k < l$ such that $\widetilde{R}_{r_k} = \widetilde{R}_{r_l}$) it is always possible to find a path with better (or equal in the worst case) benefit by deleting the regions from k to $l-1$ from the path.

Taking it into account we define the similarity degree between two regions as the benefit of the optimum path joining them.

Definition 4.5 We introduce the fuzzy similarity relation $Sim_{\widetilde{\Theta}}$ between fuzzy regions in $\widetilde{\Theta}$ to be

$$Sim_{\widetilde{\Theta}}(\widetilde{R}_s, \widetilde{R}_t) = \max_{\omega_{st} \in \Omega_{st}^{\widetilde{\Theta}}} \{ben(\omega_{st})\} \quad (11)$$

if $\Omega_{st}^{\widetilde{\Theta}} \neq \emptyset$, and 0 otherwise.

Some of the properties verified by this fuzzy similarity relation are the following:

Proposition 4.1 $Sim_{\widetilde{\Theta}}$ is reflexive.

Proof 4.1 $Res_{\widetilde{\Theta}}$ is reflexive, so for every $\widetilde{R}_k \in \widetilde{\Theta}$ there is a path $(\widetilde{R}_k, \widetilde{R}_k)$ whose benefit is 1. Hence, $Sim_{\widetilde{\Theta}}(\widetilde{R}_k, \widetilde{R}_k) = 1$.

Proposition 4.2 $Sim_{\tilde{\Theta}}$ is symmetric.

Proof 4.2 $Res_{\tilde{\Theta}}$ is symmetric, and the calculation of $Sim_{\tilde{\Theta}}$ from $Res_{\tilde{\Theta}}$ involves symmetric functions only (maximum and minimum, see equations 10 and 11) so $Sim_{\tilde{\Theta}}$ is symmetric.

Proposition 4.3 $Sim_{\tilde{\Theta}}$ is max-min transitive, i.e.

$$Sim_{\tilde{\Theta}}(\widetilde{R}_s, \widetilde{R}_t) \geq \max_{\widetilde{R}_u \in \tilde{\Theta}} \min\{Sim_{\tilde{\Theta}}(\widetilde{R}_s, \widetilde{R}_u), Sim_{\tilde{\Theta}}(\widetilde{R}_u, \widetilde{R}_t)\} \quad (12)$$

Proof 4.3 Let $\widetilde{R}_u \in \Theta$, we shall show that

$$Sim_{\tilde{\Theta}}(\widetilde{R}_s, \widetilde{R}_t) \geq \min\{Sim_{\tilde{\Theta}}(\widetilde{R}_s, \widetilde{R}_u), Sim_{\tilde{\Theta}}(\widetilde{R}_u, \widetilde{R}_t)\}$$

Let ω_{su} be the maximum benefit path between \widetilde{R}_s and \widetilde{R}_u , and let ω_{ut} be the maximum benefit path between \widetilde{R}_u and \widetilde{R}_t . Let ω_{st} be the concatenation of paths ω_{su} and ω_{ut} . Then

$$ben(\omega_{st}) = \min\{Sim_{\tilde{\Theta}}(\widetilde{R}_s, \widetilde{R}_u), Sim_{\tilde{\Theta}}(\widetilde{R}_u, \widetilde{R}_t)\}$$

Also it is obvious that $\omega_{st} \in \Omega_{st}^{\tilde{\Theta}}$, and by equation (11)

$$Sim_{\tilde{\Theta}}(\widetilde{R}_s, \widetilde{R}_t) = \max_{\omega \in \Omega_{st}^{\tilde{\Theta}}} \{ben(\omega)\} \geq$$

$$\geq ben(\omega_{st}) = \min\{Sim_{\tilde{\Theta}}(\widetilde{R}_s, \widetilde{R}_u), Sim_{\tilde{\Theta}}(\widetilde{R}_u, \widetilde{R}_t)\}$$

It is well known that any α -cut of a max-min transitive similarity relation is a crisp equivalence relation. In particular, any α -cut of $Sim_{\tilde{\Theta}}$, noted $(Sim_{\tilde{\Theta}})_{\alpha}$, is a crisp equivalence relation in $\tilde{\Theta}$. In the following section, we shall use this property to obtain a hierarchy of nested fuzzy segmentations.

4.3 Hierarchical segmentation

From $Sim_{\tilde{\Theta}}$ we shall obtain a hierarchy of nested fuzzy segmentations $H_{\tilde{\Theta}} = \{\tilde{\Theta}_1, \dots, \tilde{\Theta}_m\}$, where the segmentation associated to each level of granularity is given by the set $\tilde{\Theta}_i = \{\widetilde{R}_1^i, \dots, \widetilde{R}_{n_i}^i\}$ with $\widetilde{R}_k^i \in \tilde{\varphi}(I)$ a fuzzy region. The number of granularity levels, m , will be equal to the number of significant α levels of $Sim_{\tilde{\Theta}}$, i.e., the number of different similarity degrees between regions in $\tilde{\Theta}$ [15]. The set with all the possible values of the parameter α is:

$$\Lambda(Sim_{\tilde{\Theta}}) = \{\alpha_1, \dots, \alpha_m\}$$

where $\alpha_1 = 1$, $\alpha_m = \min\{Sim_{\tilde{\Theta}}(\widetilde{R}_s, \widetilde{R}_t) \mid \widetilde{R}_s, \widetilde{R}_t \in \tilde{\Theta}\}$, and $\alpha_i > \alpha_{i+1} \forall i \in \{1, \dots, m\}$.

The value of α indicates the similarity value between pairs of regions above which they will be joint. So if α takes value 1, no regions will be combined and if α is 0 all regions in the image will be merged into a single one.

Then, for each $\alpha_i \in \Lambda(\text{Sim}_{\tilde{\Theta}})$, a fuzzy segmentation $\tilde{\Theta}_i$ will be defined. Each fuzzy region \tilde{R}_k^i will be obtained by joining together those fuzzy regions in $\tilde{\Theta}$ that are in the same equivalence class of the quotient set $\tilde{\Theta} / (\text{Sim}_{\tilde{\Theta}})_{\alpha_i}$ [15]. Following this procedure, it is obvious that the number of regions in each level is $n_i = |\tilde{\Theta} / (\text{Sim}_{\tilde{\Theta}})_{\alpha_i}|$.

To join regions, the membership degree of a pixel to the new region is obtained as the maximum membership degree of the pixel to the regions that are being joined. More formally, let

$$\tilde{\Theta} / (\text{Sim}_{\tilde{\Theta}})_{\alpha_i} = \{C_1^{\tilde{\Theta}, \alpha_i}, \dots, C_{n_i}^{\tilde{\Theta}, \alpha_i}\}$$

Then

$$\mu_{\tilde{R}_k^i}(p_j) = \max_{\tilde{R} \in C_k^{\tilde{\Theta}, \alpha_i}} \mu_{\tilde{R}}(p_j) \quad (13)$$

For example, from the quotient set $\tilde{\Theta} / (\text{Sim}_{\tilde{\Theta}})_1$ we obtain the original fuzzy segmentation $\tilde{\Theta}$. From $\tilde{\Theta} / (\text{Sim}_{\tilde{\Theta}})_{\alpha_m}$ we obtain an unique region that corresponds to the whole image. The membership of each pixel to that single region will be the maximum membership degree of the pixel to the regions in $\tilde{\Theta}$.

Once we have defined the different levels $\tilde{\Theta}_i$, we shall show that they form a nested fuzzy hierarchy; it is well known that the α -cuts of a fuzzy relation are nested, i.e., let $i \in \{2, \dots, m\}$ be a level of $H_{\tilde{\Theta}}$ and $C_k^{\tilde{\Theta}, \alpha_i} \in \tilde{\Theta} / (\text{Sim}_{\tilde{\Theta}})_{\alpha_i}$. Then, there exists a set $\{l_1, \dots, l_q\}$ with $1 \leq q \leq n_{i-1}$ such that

$$C_k^{\tilde{\Theta}, \alpha_i} = \bigcup_{x=1}^q C_{l_x}^{\tilde{\Theta}, \alpha_{i-1}} \quad (14)$$

The following proposition follows:

Proposition 4.4 *$H_{\tilde{\Theta}}$ is a nested fuzzy hierarchy of segmentations. In particular, for every $\tilde{R}_k^i \in \tilde{\Theta}_i$ with $i \in \{1, \dots, m-1\}$ there exists $\tilde{R}_l^{i+1} \in \tilde{\Theta}_{i+1}$ such that $\tilde{R}_k^i \subseteq \tilde{R}_l^{i+1}$ (the inclusion is in the sense of lesser degree, i.e., $\mu_{\tilde{R}_k^i}(p) \leq \mu_{\tilde{R}_l^{i+1}}(p) \forall p \in I$).*

Proof 4.4 *Immediate from (13) and (14).*

5 Results

Figures 2, 3, 4 and 5 show four examples with the resulting segmentation obtained by our hierarchical approach. In all the cases, the figure contains (i) the original

image, (ii) the initial segmentation corresponding to the first stage of the algorithm, and (iii) some segmentations corresponding to different levels of the hierarchy. To display the results, the segmentation has been "defuzzified" allocating each pixel in the region for which it has the highest membership degree. By the side, a three-dimensional representation of the membership degrees corresponding to one of the fuzzy region is showed.

The image of figure 2 presents a synthetic example with four hexagons with a gradation in its colours. At the first level of the hierarchy (figure 2(B)), five regions are detected, corresponding to the background and the four homogeneous areas in the center. If the precision level is diminished, the four central areas are joint as an unique region (figure 2(C)). For the first level, figure 2(D) shows a three-dimensional representation of the membership degrees of the fuzzy region labeled as 1. Due to the degradation of colours, there is a decreasing of the membership degrees along the sequences of hexagons (from region 1 to region 4). This reveals the use of topographic information (paths between pixels) in the extraction of fuzzy regions. In a similar way, figure 2(E) illustrates the three-dimensional representation associated to the region 1 of the segmentation presented in 2(C).

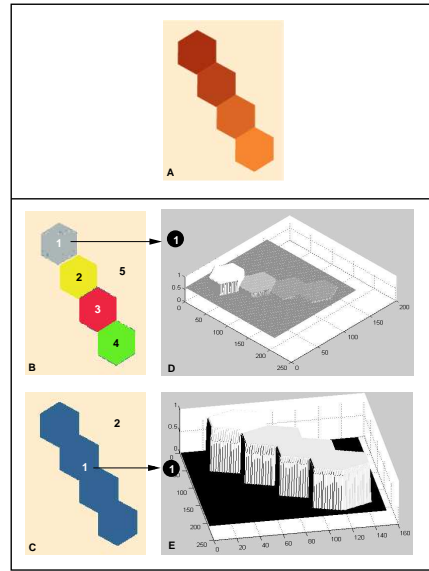


Figure 2: A synthetic example with similar regions. (A) Original image. (B) Initial fuzzy regions corresponding to the first level of the hierarchy. (C) Fuzzy regions at a high level of the hierarchy. (D) 3D representation of the membership degrees corresponding to the region 1 in (B). (E) 3D representation of the membership degrees corresponding to the region 1 in (C)

The figure 3 shows a real image where different levels of precision are detected by our approach. At the first level (figure 3(B)), several regions are extracted associated to areas with different colours (for example, into the turtle's shell). At

the level represented in figure 3(C), corresponding to the α -cut having $\alpha = 0.9$, regions of the previous stages are joint in larger regions (see the union of regions within the turtle's shell). In a similar way, figure 3(D) illustrates a higher level of the hierarchy calculated with $\alpha = 0.8$. Finally, at lower granularity levels, an unique region corresponding to the whole turtle is detected as shown in figure 3(E) (an α -cut with $\alpha = 0.75$ has been used). A three-dimensional representation of the membership degrees of the fuzzy region corresponding to the turtle is showed in figure 3(F)

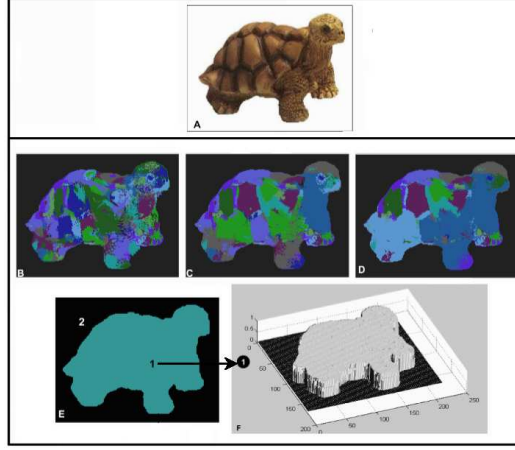


Figure 3: A real image example. (A) Original image. (B) Initial fuzzy regions corresponding to the first level of the hierarchy. (C-D) Fuzzy regions at a half level of the hierarchy. (E) Fuzzy regions at a high level of the hierarchy. (F) 3D representation of the membership degrees corresponding to the region 1 in (E).

In figure 4 there is another example of a real image where different ganularity levels may be seen. At the first level (figure 4(B)), each color area corresponds to a defuzzified region extracted from the image (see the different parts of the existing peppers). In this image all the extracted fuzzy regions are present. Figure 4(C) shows the result of an intermedium α -cut where $\alpha = 0.85$. Regions in this figure have been obtained by joining together some regions of the previous stages, into larger regions (see how the long pepper on the left of the image has been joint with its upper extreme). With an α -cut having $\alpha = 0.79$, a higher level of the hierarchy is showed in figure 4(D). In this figure more regions are joint, as can be appreciated in the round pepper's brights, in the center of the image. Finally, figures 3(E) and (F) show three-dimensional representations of the membership degrees of the regions associated to the two highest peppers in the original image, marked with numbers from 1 to 2 in figure 4(D).

Finally figure 5 presents a third example of a real image. As in the previous examples, the first level of the hierarchy, where all the fuzzy regions extracted are showed, appears in figure 5(B). Figure 5(C) shows the granularity level for an α -cut with $\alpha = 0.68$, where the union of some of the regions above house's right

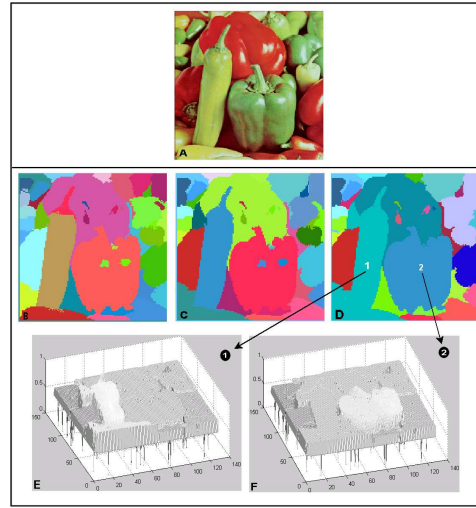


Figure 4: A real image example. (A) Original image. (B) Initial fuzzy regions corresponding to the first level of the hierarchy. (C) Fuzzy regions at an intermediate level of the hierarchy. (D) Fuzzy regions at a high level of the hierarchy. (E) 3D representation of the membership degrees corresponding to the region 1 in (D). (F) 3D representation of the membership degrees corresponding to the region 2 in (D).

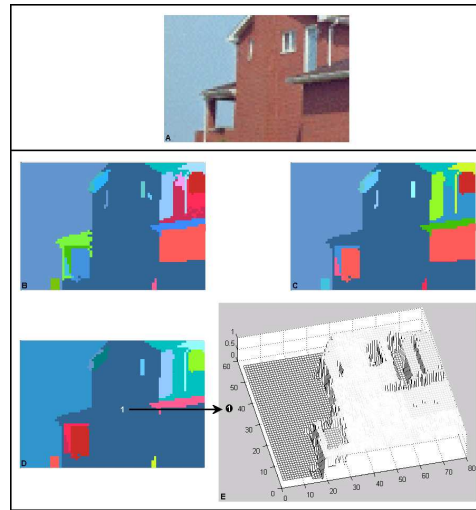


Figure 5: A real image example. (A) Original image. (B) Initial fuzzy regions corresponding to the first level of the hierarchy. (C) Fuzzy regions at an intermediate level of the hierarchy. (D) Fuzzy regions at a high level of the hierarchy. (E) 3D representation of the membership degrees corresponding to the region 1 in (C).

roof has taken place. Similarly, figure 5(D) displays the result of applying on the hierarchy an α -cut having $\alpha = 0.62$. As it can be appreciated, most of the regions that integrate the shape of the house have been joint into a single one, whose membership degrees are presented in the three-dimensional representation of the figure 5(E).

6 Conclusions

A new methodology to segment colour images by means of a nested hierarchy of fuzzy partitions has been presented. To obtain the fuzzy partition corresponding to the first level of the hierarchy, a technique based on the growth of regions has been applied. A region has been defined as a fuzzy subset of connected pixels and it has been constructed using topographic and colour information. To process the colour, a methodology based on a distance between colours defined within the $CIE(L^*a^*b^*)$ colour space has been used. For this initial segmentation, our experiments suggest the combination of the proposed colour distance and the growing region process improves the results obtained with the classical segmentation techniques.

In order to obtain different levels of granularity in the resulting segmentation, a hierarchical approach, based on a similarity relation between regions, has been employed. The similarity relation has been calculated on the basis of a resemblance one, and using the idea of path between regions. The results show that this new approach allows to consider different levels of precision when we look at the image: at high precision levels, several, small and homogeneous regions are obtained; as the precision level is diminished, less, bigger and more heterogeneous regions are achieved by joining together several regions of previous levels.

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